Caley Table $(G,*)$ $G=\{a,b,c,d,e,f,g,h\}$ $o(G)=8$

*	a	b	c	d	e	f	g	h
a	b	c	d	a	g	h	f	e
b	c	d	a	b	f	e	h	g
c	d	a	b	c	h	g	e	f
d	a	b	c	d	e	f	g	h
e	h	f	g	e	b	d	a	c
f	g	e	h	f	d	b	c	a
g	e	h	f	g	c	a	b	d
h	f	g	e	h	a	c	d	b

Possible orders (Lagrange): 1, 2, 4, 8

Element orders:

$o(a)=o(c)=4$

$o(b)=2$ Autosym.

$o(d)=1$ Neutral

$o(e)=o(f)=4$

$o(g)=o(h)=4$

$\left.\begin{array}{l} a*a=b \\ a*a*a=c \\ a*a*a*a=d \end{array}\right\} \rightarrow o(a)=4$

The law is internal ✓

$\exists!$ neutral element $\rightarrow d$ ✓

Every element has its symmetrical:

a, c ✓ Sym. e, f ✓ Sym.

b ✓ Autosym. g, h ✓ Sym.

d ✓ Autosym.

Possible subgroup orders (Lagrange): 1, 2, 4, 8

$S_1=\{d\}$ $o(S_1)=1$

$S_2=\{a,b,c,d,e,f,g,h\}$ $o(S_2)=8$

$S_3=\{d,b\}$ $o(S_3)=2$

$S_4=\{d,b,a,c\}$ $o(S_4)=4$

$S_5=\{d,b,e,f\}$ $o(S_5)=4$

	d	b	e	f
d	d	b	e	f
b	b	d	f	e
e	e	f	b	d
f	f	e	d	b

$S_6=\{d,b,g,h\}$ $o(S_6)=4$

*	a	b	c	d	e	f	g	h
a	b	c	d	a	g	h	f	e
b	c	d	a	b	f	e	h	g
c	d	a	b	c	h	g	e	f
d	a	b	c	d	e	f	g	h
e	h	f	g	e	b	d	a	c
f	g	e	h	f	d	b	c	a
g	e	h	f	g	c	a	b	d
h	f	g	e	h	a	c	d	b

A groups elements must ALWAYS be ASOCIATIVE

$\forall \alpha, \beta, \gamma \in G$

$\alpha * (\beta * \gamma) = (\alpha * \beta) * \gamma$ ✓

The group is NOT ABELIAN e.g. → $h*a=f \neq a*h=e$

$S_1 = \{d\}$

$S_2 = \{a,b,c,d,e,f,g,h\}$

The MAX and MIN order Subgroups are NORMAL by definition

$S_3 = \{d, b\}$ ✓ Normal

A subgroup H is NORMAL $\iff \forall x \in G \quad x*H = H*x$

$S_4 = \{d, b, a, c\}$ ✓ Normal

$S_5 = \{d, b, e, f\}$ ✓ Normal

$S_6 = \{d, b, g, h\}$ ✓ Normal

Example: lets see if $e * S_4 = S_4 * e$

$$e * S_4 = \{e*d, e*b, e*a, e*c\} = \{e, f, h, g\}$$

$$\parallel$$

$$S_4 * e = \{d*e, b*e, a*e, c*e\} = \{e, f, g, h\}$$

G/H - partitioned groups

A NORMAL subgroup can create a partition in its group through the following

EQUIVALENCE RELATION : $\forall \alpha, \beta \in G \;/\; \alpha R \beta \iff \alpha * \beta' \in H$

$$\bar{\alpha} = \{\gamma \in G \,/\, \alpha R \gamma\}$$

$$\left.\begin{array}{l} \alpha * \beta' \in H \to \alpha * \underbrace{\beta' * \beta}_{\text{Neutral}} \in H * \beta \to \alpha \in H * \beta \\ \alpha \in \bar{\beta} \end{array}\right\} \bar{\beta} = H * \beta$$

since H is a NORMAL SUBGROUP

$$\bar{\beta} = \beta * H$$

- H performs a partition on $G \longrightarrow G/H$
- The elements of G/H are the Equivalence Classes $\rightarrow \bar{x} \in G/H$

$$\boxed{\bar{x} = x * H = H * x}$$

$\forall \bar{x}, \bar{y} \in G/H \qquad \bar{x} * \bar{y} = \overline{x * y}$

$\forall\, \gamma$ Neutral element of $G \longrightarrow \bar{\gamma}$ is the neutral element of G/H

*	a	b	c	d	e	f	g	h
a	b	c	d	a	g	h	f	e
b	c	d	a	b	f	e	h	g
c	d	a	b	c	h	g	e	f
d	a	b	c	d	e	f	g	h
e	h	f	g	e	b	d	a	c
f	g	e	h	f	d	b	c	a
g	e	h	f	g	c	a	b	d
h	f	g	e	h	a	c	d	b

$S_1 = \{d\}$ ✓ Normal

$S_2 = \{a, b, c, d, e, f, g, h\}$ ✓ Normal

$S_3 = \{d, b\}$ ✓ Normal

$S_4 = \{d, b, a, c\}$ ✓ Normal

$S_5 = \{d, b, e, f\}$ ✓ Normal

$S_6 = \{d, b, g, h\}$ ✓ Normal

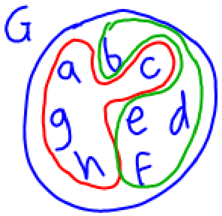
G/S_5

$\bar{a} = a * S_5 = \{a, c, g, h\} = \bar{c} = \bar{g} = \bar{h}$

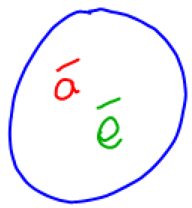
$\forall \bar{x}, \bar{y} \in G/H \quad \bar{x} * \bar{y} = \overline{x * y}$

$\bar{e} = e * S_5 = \{e, f, b, d\} = \bar{f} = \bar{b} = \bar{d} = S_5$ Neutral element of G/S_5

G



G/S_5



*	$\bar{a}$	$\bar{e}$
$\bar{a}$	$\bar{e}$	$\bar{a}$
$\bar{e}$	$\bar{a}$	$\bar{e}$

G/S_3 $\quad S_3 = \{d, b\}$

$\bar{a} = a * S_3 = \{a, c\} = \bar{c}$

$\bar{b} = b * S_3 = \{b, d\} = \bar{d} = S_3 \equiv$ Neutral element G/S_3

$\bar{e} = e * S_3 = \{e, f\} = \bar{f}$

$\bar{g} = g * S_3 = \{g, h\} = \bar{h}$

$*$	$\bar{a}$	$\bar{b}$	$\bar{e}$	$\bar{g}$
$\bar{a}$	$\bar{b}$	$\bar{a}$	$\bar{g}$	$\bar{e}$
$\bar{b}$	$\bar{a}$	$\bar{b}$	$\bar{e}$	$\bar{g}$
$\bar{e}$	$\bar{g}$	$\bar{e}$	$\bar{b}$	$\bar{a}$
$\bar{g}$	$\bar{e}$	$\bar{g}$	$\bar{a}$	$\bar{b}$

Abelian ✓